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Virtual Pitch and the Classification of Chords in Minor and Major Keys

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ABSTRACT

This paper makes use of the virtual pitch model as developed by Hofmann-Engl in order to demonstrate that Hugo Riemann's functional harmonic system has a psychological basis and that is can be considered to be superior to the Roman numeral system.

It will be shown that the comparison of tonal chords within a given key, produces high similarities between the set of virtual pitches of chord I to VI, of chord IV to II and of chord V to III. Additionally, the tension between chord I and V can be explained by its highest dissimilarity. This adds momentum to Riemann's terminology by relating the tonic (I) to the parallel tonic (VI), the subdominant (IV) to the subdominant parallel (II) and the dominant (V) to the dominant parallel (III). The closing effect of the perfect cadence can be seen as an effect of tension and resolution.

These results act in a dual fashion. They add further support to the validity of the Hofmann-Engl pitch (virtual pitch according to Hofmann-Engl), and suggests at the same time that the international community might profit from adapting Riemann's system.

I. INTRODUCTION

Based on the concept of residual pitch as introduced by Schouten (1940), Terhardt (1977, 1977, 1979) extended this concept to what he called virtual pitch. This is, while residual pitch refers to the phenomenon where the fundamental frequency of a pitch signal is not present but will nevertheless be heard, virtual pitch on the other hand refers to a pitch which might not be heard but which the frequency implies and is closely related to the idea of Rameau's (1722) of the roots.

During the eighties and nineties, several attempts were undertaken in order to establish a workable model, or let's say a virtual pitch predictor. However, as pointed out by Hofmann-Engl (2004, 2006), none of these models came without major problems and up-to-date, it seems that the Hofmann-Engl pitch has remained unchallenged and has been implemented in the context of contemporary composing (Hofmann-Engl, 1999) and the analysis of contemporary music (Hofmann-Engl, 2004).

In this paper, now the Hofmann-Engl pitch will be used in order to compare two systems of chord notation. The first system, the Roman numeral system is internationally in use. The other system however, the Riemann (1880) or *functional harmony* system appears to be almost exclusively in use within the German speaking countries. The paper will show that, while the Roman numeral system leaves the chords to a given chord unrelated by simply numbering them, the Riemann system clusters the six main chords into three groups and recognizes the tension between chord I (tonic) and chord V (dominant) as a central feature of tonal music.

While the paper will firstly introduce elements of the Hofmann-Engl pitch, it then will focus on how the similarity of two chords can be computed by correlating their respective virtual pitch spectra. Subsequently, the paper will introduce briefly both the Roman numeral and the Riemann system. By computing the similarity of the six main chords of a key, it will be shown the classification according to Riemann has a psychological basis. In a final section the paper will make a case for the Riemann system to be used as an international standard.

II. The Hofmann-Engl Pitch

For practical reasons, we refrain from using the term *Hofmann-Engl's virtual pitch model* and replace it by the term *Hofmann-Engl pitch*.

The *Hofmann-Engl pitch* was first introduced in 1990 by Hofmann-Engl and has been use in a number of contexts by the same author (1999, 2004 & 2006). Not dissimilar to Terhardt's virtual pitch model, the *Hofmann-Engl pitch* is a methof by which a given chord is scanned along its overtone series and a number of fundamental frequencies (roots) are extracted according to best match, second best match, third best match ect. We will give an example.

The cord *c*, *e*, *g* consists of three different tones. The tone *c* is the first component of the overtone series on *c*, the second component of the overtone series based on *f*, the third component of the overtone series based on a^b and so on. Similarly the tone *e* is part of the spectrum of *e,* of the spectrum based on *a*, the spectrum based on *c* and so on. All in all, we obtain for the three notes of the c-major chord the following table:

Table 1. The ordered overtone series containing the tones *c*, *e* and *g*

Overtone number	c	e	g	Value for *b*
1	*C*	*E*	*G*	0
2	*F*	*A*	*C*	1
4	A^b	*C*	E^b	2

6	*D*	*F#*	*A*	3
8	B^b	*D*	*F*	4
14	D^b	*F*	A^b	5

Note that this table is similar to Terhardt's table from 1982, but has been extended to include the 14th overtone. The motivation for Terhardt to choose the cut-off point with the overtone number 8 is to avoid ambiguity. This is, allowing for the cut-off point at the 14th overtone renders both the *c* and the *f* roots at first degree according to Terhardt. However, this ambiguity can be overcome by using the weights as introduced by Hofmann-Engl (1990).

The first weight used is:

(1) $$w_f = \frac{c^2 - b^2}{c}$$

With w_f as the fusion weight, *c* as the constant with *c* = 6 *Hh* and *b* as the place of the tone within the table, with $b_1 = 0$ *Hh*, $b_2 = 1$ *Hh*, $b_3 = 3$ *Hh* ect.

For instance, the *c* under the tone *c* fetches the value (36 – 0/6) *Hh* = 6 *Hh* , the *f* the value (36 – 1)/6 *Hh* = 5.83 *Hh,* the a^b the value (36 – 4)/6 *Hh* = 5.33 *Hh* ect.

The second weight adds more weight to lower notes and less weight to higher notes. The expression is:

(2) $$w_p = \sqrt{\frac{1}{i}}$$

With w_p as the weight according to pitch order, and *i* the number of the note within in the chord according to low to high pitch

In our example above (chord c, e, g) we get the weight for *c* of 1, for *e* the weight 0.71 and for *g* the weight 0.58.

Applying these weights, we obtain the following Hofmann-Engl pitch set to the *c-major* chord (*c*, *e*, *g*):

Table 2. Strength of the *Hofmann-Engl pitch* for the *c-major* chord

Chord: c-major (root position)	
strength of *c* is 4.37 Hh	strength of *g* is 1.15 Hh
strength of *f* is 3.01 Hh	strength of *a#* is 1.11 Hh
strength of *d* is 2.28 Hh	strength of *f#* is 1.06 Hh
strength of *a* is 2.24 Hh	strength of *d#* is 1.02 Hh
strength of *g#* is 2.13 Hh	strength of *c#* is 0.61 Hh
strength of *e* is 1.41 Hh	-

As expected, c fetches the strongest *Hofmann-Engl* pitch and thus can be considered to be the root to the *c-major chord in root position.*

For a more in depth discussion on the *Hofmann-Engl pitch*, the reader might be referred to Hofmann-Engl (1999, 2004 & 2006). A fully functional applet with user friendly *GUI* can be found here:

www.chameleongroup.org.uk/software/piano.html

III. Calculating Chord Similarities

Each chord based on the 12-tone equal temperament note system generates a set of *Hofmann-Engl pitches* of 6 to 12 tones. If a chord such as the chord *c*, *c* and *c* generates s less than 12 *Hofmann-Engl pitches* only, we set the value for those non-generated *Hofmann-Engl* pitches 0. This allows us to map every chord onto a set of 12 *Hofmann-Engl pitches*. We will refrain from a offering a formal definition, but will illustrated the concept within an example. Computing the *Hofmann-Engl pitch* set for the *c-minor* chord, we obtain:

Table 3. Strength of the *Hofmann-Engl pitch* for the *c-minor* chord

Chord: a-minor (root position)	
strength of *f* is 3.64 Hh	strength of *b* is 1.25 Hh
strength of *g#* is 3.50 Hh	strength of *g* is 1.15 Hh
strength of c is 3.12 Hh	strength of *a#* is 1.11 Hh
strength of d# is 2.44 Hh	strength of *a* is 0.86 Hh
strength of d is 1.50 Hh	strength of *e* is 0.43 Hh
strength of c# is 1.39 Hh	-

Ordering the *Hofmann-Engl pitch* set for both the *c-major* and *c*-minor chord along the chromatic scale *c*, *c#*, d …, *b* we obtain the following table:

Table 3. Hofmann-Engl pitch sets for c-major and c-minor

Hofmann-Engl Pitch	c-major	c-minor
c	4.37 Hh	3.12 Hh
c#	0.61 Hh	1.40 Hh
d	2.28 Hh	1.50 Hh
d#	1.02 Hh	2.44 Hh
e	1.41 Hh	0.43 Hh
f	3.01 Hh	3.65 Hh
f#	1.06 Hh	0.00 Hh
g	1.15 Hh	1.15 Hh
g#	2.13 Hh	3.50 Hh
a	2.24 Hh	0.87 Hh

a#	1.11 Hh	1.11 Hh
b	0.00 Hh	1.25 Hh

A great number of similarity measures have been used over the last decade (compare: Hofmann-Engl, 2005), but at this instant, the author suggest to simply compute the correlation coefficient between the two *Hofmann-Engl* pitch sets starting with *c* and ending with *b*. Here, we obtain a correlation with r = 0.59. Now, in order to check whether this is a sensible result, we compare the c-major chord with the c#-major chord by computing the correlation between the relevant *Hofmann-Engl pitch* sets expecting a much smaller correlation coefficient. Indeed, computing the correlation coefficient for both sets results in r = 0.30. This is not to say, that the correlation coefficient is the best similarity predictor for computing the similarity between chords, but, so the author argues, it seems to be sufficient for our purposes.

IV. Chord Classification Systems

The six main chords within a given key are the illustrated on a stave below:

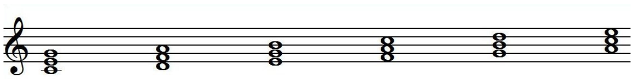

Figure 1. The six main chords in the key of c-major

This is, choosing the *c-major* key, we obtain the *c-major*, the *d-minor, e-minor*, *f-major, g-major* and *a-minor* chords.

A. The Roman Numeral System

The Roman numeral system is possibly the most used system in the world, and is essentially simple as the following stave might illustrate:

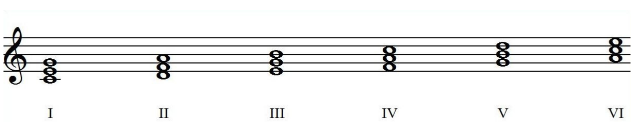

Figure 2. The Roman numeral system in c-major

This system is self-explanatory as the chords have simply been numbered with Roman numerals and this might also explained its wide-spread use. However, this system does not make any functional connections, and this is exactly what Riemann (1880) considered a deficiency and hence, he proposed his own system, often called functional harmony system or the *Riemann* system.

B. The *Riemann system*

Inspired by the investigations of Helmholtz into acoustics and psycho-acoustics, Riemann believed that the Roman numeral system did not capture the fact that chord sequences such as perfect, interrupted and imperfect sequences made somehow *sense*. Thus, Riemann replaced the numerals with names relating chords to each other or let's say construction functional relationships. The new system replaced the numbers in the following fashion:

- I with ***Tonic***
- II with ***Subdominant Parallel***
- III with ***Dominant Parallel***
- IV with ***Subdominant***
- V with ***Dominant***
- VI with ***Parallel Tonic***

Not only did Riemann thus cluster the 6 chords into 3 groups, but he observed that the effect of the prefect cadence need to be understood as being based upon some sort of tension, and because the subdominant is the dominant to the tonic, the cadence I – IV – V – I could be interpreted as: I – IV resolves a tension and V builds up a tension which gets resolved by returning to I. However, to underpin Riemann's concept with more accurate psychological means, was at his time not possible. Thus, the *Hofmann-Engl pitch* is supposed to shed light onto the question whether Riemann's assumptions were meaningful.

V. Similarity Ratings of Chords within a Key

In order to obtain a clearer idea about how similar the six chords within a major/minor (referring here to a natural minor) key, we computed the correlation coefficients of the *Hofmann-Engl pitch* sets of each chord, comparing each of the 6 chords with each other. The following matrix was obtained:

Table 4. Hofmann-Engl pitch similarity matrix for the 6 main chords of a major (minor) key based on correlation coefficiants

	I	II	III	IV	V	VI
I	1	0.05	0.17	0.35	0.03	0.37
II	0.05	1	0.11	0.65	0.38	0.37
III	0.17	0.11	1	0.36	0.65	0.20
IV	0.35	0.65	0.36	1	0.05	0.38
V	0.03	0.38	0.65	0.05	1	0.00
VI	0.37	0.37	0.20	0.38	0.00	1

As mentioned above, computing chord similarities by making use of correlation coefficients along the chromatic scale might not be the ultimate method, but it clearly produces already convincing results.

Firstly, we find that chords IV and II are highly similar (0.65) as are chords V and III (0.65). Hence, using the terminology of subdominant (IV) and subdominant parallel (II) alongside the terms dominant (V) and dominant parallel (III) make perfect sense.

It further shows the closeness of the *tonic* to the *subdominant*, but it also shows that the *tonic* closed to chord VI, Riemann's *parallel tonic.*

Finally, the very low similarity (0.03) between the *tonic* and the *dominant,* the minimal similarity (0.00) between the *dominant* and the *parallel tonic* and the very low similarity (0.05) between *tonic* and *dominant parallel*, do not only support the classification, but explain the tension between *tonic* chords and *dominant* chords and the effect of resolution they produce in the order *dominant – tonic.*

VI. CONCLUSION

This paper set out to compare two systems which describe chords within a tonal setting. One of these systems is the generally used Roman numeral system and the other the less well known Riemann system which clusters the chords into 3 groups and relates those groups via tension and resolution. However, at the time, when Riemann made his observations, no particular psychological model was available in order to confirm these observations.

This paper made use of Hofmann-Engl's virtual pitch model which was coined the *Hofmann-Engl pitch* in order to compute similarity ratings between the 6 main chords of a major/minor key. There, it was found that the clustering of chords into *tonic, subdominant* and *dominant* chords could be validated by referring to the similarity ratings the chords fetched by computing the correlation coefficients of respective chords.

This paper concludes that the Riemann system is superior to the Roman numeral system and that it might be beneficial to adapt it more widely.

It finally indicates that the *Hofmann-Engl pitch* might provide the basis for powerful chord similarity ratings which could prove useful in audio-compression process, music information retrieval, musical analysis and composition.

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