Virtual pitch and pitch salience in contemporary composing

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1 INTRODUCTION

The beginning of this century opened a debate in composition theory, which too a large extent remains unresolved. While there was a polarization at first, between the schools around Hindemith and Schoenberg, the first school lost its influence almost entirely in the 60s and the second gained great momentum engendering serialism and experimental music in the 70s. Even still in the 80s many composers who had not embarked on either serialism or experimental music found themselves alienated and isolated. The climate changed over the last decade and the motto "anything goes" is widespread. However, the debate itself does not receive great attention. The fact that Hindemith's scientific approach was exposed by Schole (1938) as pseudoscientific strengthened the arguments of the opponents who understood music as a solely socio- cultural phenomenon (Adorno 1949). This also might explain why psycho- acoustic research has been marginalized and only few composers have a more comprehensive understanding of the subject matter. In this context, Terhardt's (1976) theory of virtual pitch takes in a prominent place. Although widely quoted and certainly of interest to composers, it still is frequently dismissed as "nonsense" even in recent years (e.g. Eberlein 1994). This is the more striking as there have been many experimental data supporting Terhardt's theory (e.g. Parncutt, 1989). It is the purpose of this paper to present a modified version of Terhardt's theory, which has been tested in an experimental setting, and to illustrate its application in three compositional examples provided by the author.

2 THE DATA

Classical harmony theory (Rameau, 1722) states that the fundamental bass of the C- major and C- minor triad is *c*. However, according to the theory of virtual pitch there exists a series of possible fundamental basses to any given chord. In case of the C- major and C- minor chord we obtain the following data according to the modified algorithm of virtual pitch:

C-major triad

c =	4.37 Hh	e =	1.14 Hh
f =	3.01 Hh	g =	1.15 Hh
d =	2.28 Hh	a# =	1.11 Hh
a =	2.24 Hh	f# =	1.06 Hh
g# =	2.13 Hh	d# =	1.10 Hh

Sonance: S = 0.467 Sh

C-minor triad

f =	3.35 Hh	d# =	1.41 Hh
g# =	3.15 Hh	c# =	1.39 Hh
c =	3.02 Hh	b =	1.25 Hh
d =	2.14 Hh	a# =	1.11 Hh
e =	1.58 Hh	f# =	0.86 Hh

Sonance S = 0.222 Sh

For the C- major chord, we find *c* = 4.37 *Hh* (*Hh* is Helmholtz and is the unit denoting the strength of c to function as fundamental bass). Comparing this value with all other values of the C- major chord, we find that *c* is the strongest fundamental bass. However, the pitches *f*, *d*, *a* ... also show some degree to function as fundamental basses, although with diminishing strength with *c#* being the weakest (least suitable to be fundamental bass). Thus, we find that these data seem not to conflict with classic harmony theory. However, the situation is changed if we consider the C-minor chord. Here, we find that the pitch *f* (= 3.35 *Hh*) is the strongest fundamental bass and not as expected *c* (= 3.02 *Hh*). Now, we are so used to hear the C-minor chord together with the bass note c that it might be difficult to hear a C-minor chord with f as fundamental bass as being more coherent. But if we try to take a naive position pretending never to have heard any chord, we might find that f merges with C-minor better indeed.

A crucial difference between classic harmony theory and virtual pitch is, that, although classic harmony theory underwent a development, it can be applied to a very limited amount of chords while the theory of virtual pitch can applied to any chord. We give to examples: The chords *c, c#, f#*, which we will call Webern triad, and the chord *c, c#, d*, which we will call *Cluster triad*.

Webern triad

d	=	2.95 Hh	b	=	1.90 Hh
g#	=	2.64 Hh	a	=	1.25 Hh
f#	=	2.52 Hh	a#	=	1.11 Hh
c#	=	2.02 Hh	d#	=	1.06 Hh
c	=	1.94 Hh	e	=	0.64 Hh
f	=	1.94 Hh	g	=	0.35 Hh

Sonance: S = 0.158 Sh

Cluster triad

d	=	3.08 Hh	d#	=	1.41 Hh
c	=	2.64 Hh	f#	=	1.37 Hh
a#	=	2.13 Hh	a	=	1.25 Hh
c#	=	2.02 Hh	g	=	1.12 Hh
f	=	1.94 Hh	e	=	0.86 Hh
g#	=	1.77 Hh	b	=	0.78 Hh

Sonance S = 0.170 Sh

The strongest tonic in both cases is d. Surprisingly at first, this seems even sensible referring back to classical harmony theory. The D-major-7 is given by: d, f#, a, c and the D-major+7 is given by: d, f#, a, c#. Combining both chords and omitting the root d and the fifth a, we obtain the Webern triad. Omission of the third f# and the fifth a, results in the cluster triad. The tone d is the fundamental bass in both cases. However, d is the stronger fundamental bass for the cluster triad (d = 3.08 Hh) compared to the Webern triad (d = 2.95). This is one of the factors, which leads to the difference in sonance. Sonance, hereby, is the degree of consonance/dissonance of a chord (related to pitch salience). A pure sine wave will fetch the value S = 1 Sh (Sh is short for Shouten). White noise will fetch the value S = 0 Sh. The fact, that the cluster triad has a higher sonance (S = 0.17) than the Webern triad (S = 0.158) means, that the Webern triad is more noise-like.

3 APPLICATIONS

3.1 Functional Relationships

For hundreds of years (since late Renaissance) composers have made use of functional relationships between chords. Of all those relationships the ones between tonic, subdominant and dominant are most prominent. However, at the early 20th century most major composers found the usage of these chordal relationships meaningless and banished then from their compositions. However, by referring to the concept of virtual pitch and pitch salience, we might find that functional relationships can gain new meaning.

The example given is taken from Cyclone (Hofmann-Engl, 1994). The piece opens with *e#*, *f#* and *b* in the right and *b*, *d* in the left hand. The strongest fundamental bass for this chord is *g* (= 3.07 *Hh*) without being present in the chord itself. In bar 4 the piece moves to a new center, which is reached in bar 5. The new chord consists of the notes: *d*, *a*, *b#*, *c#* and *a*. Strongest fundamental bass now is d (= 3.24 *Hh*). The subdominant *g* leads to the tonic *d*. Finally, in bar 10 we reach the dominant *a* on the first beat followed by the chord: f#, e, *a*, e^b. Although our model predicts *b* (= 2.77 *Hh*) to be the strongest and *d* (= 2.76) to be the second strongest fundamental bass, it is likely that d will be perceived as the strongest in this context. However, the sonance of the chord (*S* = 0.165) is rather small, the progression from dominant to tonic will be perceived as an imperfect close - the piece goes on.

Piece No. 6 from the composition *Cyclone* (Hofmann-Engl, 1994), illustrating the use of functional harmony in context of virtual pitch. A recording of the piece is available on: http://freespace.virgin.net/ludger.hofmann-engl/biography.html

3.2 Virtual Tonality

Although tonality, as defined by Fétis (1844) represents nothing than the syntactic level of a composition (thus atonal music is as tonal as classical tonal music is), it soon was exclusively used to describe classical tonality. The concept of what we will call virtual tonality deviates fundamentally from classical understanding.

This time, the example is taken form the 2nd movement of the 5th piano sonata (Hofmann- Engl, 1993):

Bars 8 to 11 form 2nd movement of the 5th piano sonata (Hofmann-Engl, 1993). Chord 1 to chord 7 are from 4th beat bar 8 to beat 1 in bar 9.

Between bar 8 last beat and bar 10 first beat, we can observe a progression of 7 chords. The bass line is: *f#, e, d#, b,* c#, *d#, b* a line which is part of the B-major scale. The table below lists the strongest fundamental bass for each chord together with the sonance:

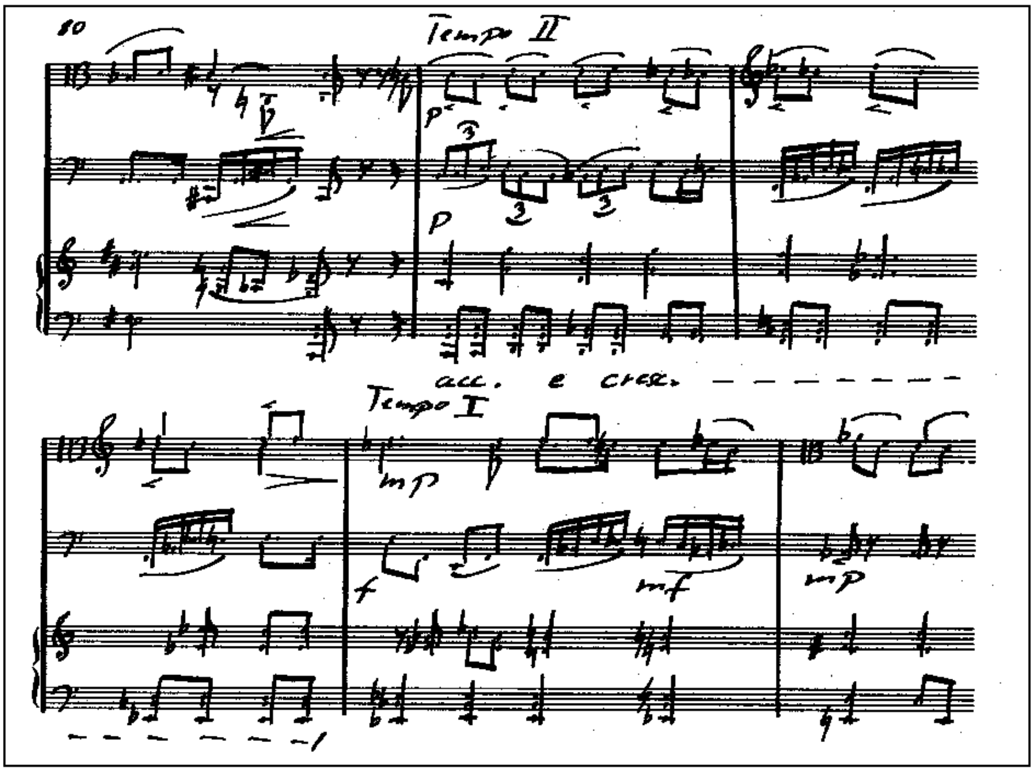
Chord 1:	b = 2.40 Hh,	S = 0.163 Sh	Chord 5:	f# = 3.75 Hh,	S = 0.498 Sh
Chord 2:	a = 3.29 Hh,	S = 0.330 Sh	Chord 6:	f = 3.29 Hh,	S = 0.307 Sh
Chord 3:	g# = 3.35 Hh,	S = 0.359 Sh	Chord 7:	g = 3.07 Hh,	S = 0.259 Sh
Chord 4:	g = 3.07 Hh,	S = 0.266 Sh			

The fundamental basses of these 7 chords form a simple melodic line but do not coincide with the bass, although they are likely to be of perceptual relevance. Further, the highest sonance is reached on chord 5 (*S* = 0.498 *Sh*). A crescendo and the fact that *f#* is the highest pitch, supports this development, thus dynamics, melody and sonance are correlated.

However, this is not what establishes virtual tonality. In order to determine the virtual tonality of the passage, we have to compare the mean strengths of all possible fundamental basses. For instance: The strength of the fundamental bass f# of the first chord is 2.4 Hh. The strength of f# of the second chord (not listed above) is 1.71 Hh. Adding the strengths of all fundamental basses f# gives us 9.84 Hh and the mean 1.4 Hh. However, computing the mean strength for all possible fundamental basses reveals that b yields the highest mean with 2.04 Hh. Thus b can be considered to be the tonal center of this passage. This is, what is meant by virtual tonality.

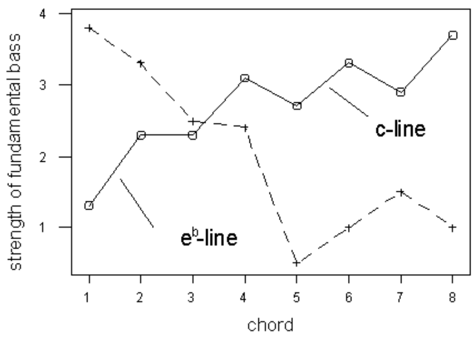
3.3 Virtual modulation

In classic harmony theory modulation refers to the change of key (e.g. changing C-major to G-major). Virtual modulation is a concept whereby the relationship between two possible fundamental basses is systematically changed. This time, the example is taken form Abstract I (Hofmann- Engl, 1992) 3rd movement:



Bar 80 to 84 taken from *Abstract I* (Hofmann-Engl, 1992) 3rd movement for viola, bassoon and harpsichord. The 9 chords are in the harpsichord (lower system) from bar 82 first beat to bar 83 first beat.

Without considering the theory of virtual pitch, we can see that the first chord has the fundamental bass *c*, while the last chord has the fundamental bass e^b. However, the question remains what happens in between. The strengths of the fundamental basses *c* and e^b over the chord sequence are plotted in the graph.



The strength of the fundamental basses *c* and e^b along the sequence of the 9 chords (chord 7 and 8 are equal and illustrated as one point 7) taken form *Abstract I*.

We find that the strength of c increases while the strength of e^b decreases, both in form of a wavy line. The sequence of the strongest fundamental basses (*c*, *c*, *g*, e^b,

b^b, e^b, e^b, e^b) confirms that a virtual modulation occurred using the mediant relationship between *g* and e^b. We are familiar with these kind of results, the method, however, is new.

4 The Algorithm

The algorithm is, as mentioned, earlier, based on the concept of virtual pitch. We will outline this concept and explain some aspects of the algorithm.

Principal idea is, if we take the tone c for instance, this *c* could be part of various overtone- serieses *c* is part of the overtone- series based on *c*. Further, the overtone- series of f includes *c*, so does a^b, *d*, b^b and d^b. Based on various arguments as put forward by C. Stumpf (1965) E. Terhardt (1982) *c* is the strongest virtual pitch to *c*, the second strongest virtual pitch is f and so on. In general, we obtain six candidates, which could serve as virtual pitches to any given tone. These are: The tone itself, the fifth down, the major third down, the minor seventh down, the major second down and the major seventh down. The strengths decreases in this order.

Precisely, this system is to be used for chords. For each tone of a given chord, we write its virtual pitches in a column. In the instance of the C- major chord we obtain the table below:

sound	c	e	g	value b
candidate 1	c	e	g	0
candidate 2	f	a	c	1
candidate 3	a^b	c	e^b	2
candidate 4	d	f#	a	3
candidate 5	b^b	d	f	4
candidate 6	d^b	f	a^b	5

In the first column, we find the virtual pitches for c, in the second column the pitches for *e* and the pitches for *g* in the third column. The last column (value *b*) gives each virtual pitch a value (a pitch not listed in a column gets the values 6). However, early experiments (Hofmann- Engl, 1990) showed that the value *b* as such is not suitable to estimate the strength of a candidate. The strength can be estimated by the formula:

$$S(c) = \frac{g^2 - b(c)^2}{g}$$

where $g = 6$ and $b(c)$ is the value of a candidate as shown in the table above and $S(c)$ is the strength of the candidate.

According to this formula the strength of the candidates is as follows (from 1 to 6): 6 Hh, 5.83 *Hh*, 5 *Hh*, 4.5 *Hh*, 3.3 *Hh* and 1.83 *Hh* (*Hh* is short for Helmholtz). A candidate not listed gets 0 *Hh*.

Considering the table again, we find that all tones *c*, *e* and *g* support the virtual pitch *c* as candidates 1, 3 and 2 with the mean 5.2 *Hh*. However, the tones c, e and g (candidates 2, 6 and 5) also support the virtual pitch f. Thus we get the smaller mean 3.7 *Hh*. Hence, we assume that c is a stronger fundamental bass then *f*.

If we intend to determine the strength of a fundamental bass (= virtual pitch) in general, we have to consider one more factor. The lower a tone within a chord, the stronger its impact on the fundamental bass. Thus, for instance, the interval *c*, *g* supports the virtual pitch *c* more then does the interval *g*, *c*. The position of a tone within a chord will be taken into account by $(1/i)^{1/2}$. Thus we get:

$$S(p) = \frac{\sum_{i=1}^{n} \left(g^2 - b_i(p)^2\right) \cdot \sqrt{\frac{1}{i}}}{n\ g}$$

where $S(p)$ is the strength of the pitch p to be fundamental bass of the chord consisting of *n* tones, *i* the place of the tone within the chord (with the lowest tone at place 1 up to the highest tone at place *n*), g = 6 Hh. and $b_i(p)$ is the value of the candidate of the tone t_i within the chord at place *i*.

Applying this formula to the C-major chord we obtain the strength of *c* as fundamental bass: $[(6^2 - 0^2)*1/1^{0.5} + (6^2 - 2^2)*1/2^{0.5} + (6^2 - 1^2)*1/3^{0.5}] / (3*6) = 4.38$. This is the number as stated earlier.

The formula for the calculation of the sonance is more complex and partly a result of the input of experimental data. However, there are two major factors, which determine its shape. Firstly, the assumption is, that the stronger the strongest fundamental bass, the higher the degree of sonance and secondly, the more virtual pitches a chord supports the smaller the degree of the sonance. The formula is:

$$S(ch) = \frac{n}{\sum_{i=1}^{n} \sqrt{\frac{1}{i}}} \quad \frac{S(p)_{max}}{k + \frac{m^2}{k} \sqrt{1 - \left(\frac{\frac{S(p)_{max}}{\sum_{j=1}^{m} S(p)_j}}{c_p}\right)^2}}$$

where $S(ch)$ is the sonance of the given chord, n the amount of tones of the chord, *i* the place of a tone within the chord, $k = 6\ Hh/Sh$, m the amount of virtual pitches of the chord, $S(p)_{max}$ the strength of the strongest virtual pitch, $S(p)_j$ the strength of the *j*th virtual pitch and $c_p = 0.224$ (the maximal possible strength of a virtual pitch in percentage).

Now, we are able to calculate the sonance for the C-major chord: $S(p)_{max} = 4.37\ Hh$, $m = 11$, $n = 3$, the sum of all $S(p)_j$ is 20.39, the sum of $(1/i)^{0.5}$ is 2.3, thus we get $S(ch) = 3/2.3*4.37/(6 + 121/6*(1 - (4.37/20.39)/0.224)^2) = 0.465\ Sh$ as stated before.

5 The experimental back-up of the algorithm

It is impossible to summarize the exact experiment design, procedure and results. However, the correlation between predictions based on the algorithms has been shown (Hofmann-Engl, 990) to be 89 % ($p < 0.001$) for the algorithm on virtual pitch and 83% ($p < 0.001$) for the algorithm on sonance. The experiment was conducted over a sample of 73 participants including high school students, undergraduate music students and postgraduate music students. It is interesting to note that the algorithm failed where minor thirds have been involved (listeners overestimate the sonance of minor thirds).

References

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